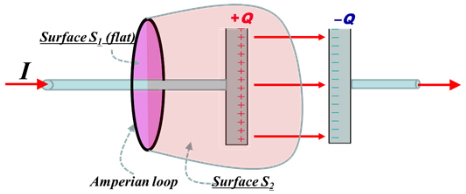
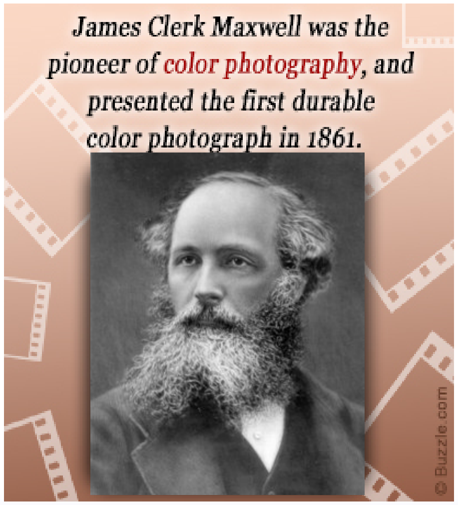
Lecture 18

University of Massachusetts
UMASS Lowell
Physics II
07.31.2015

Chapter 34



Maxwell's equations



$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

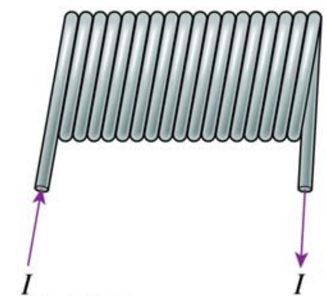
$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Lecture Capture:
http://echo360.uml.edu/danylov201415/physics2spring.html

Inductors

Inductors (solenoids) store potential energy in a form of a magnetic field.

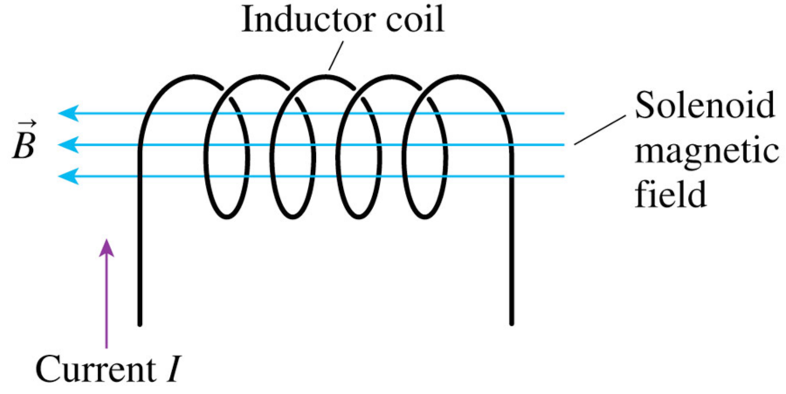


Inductance (definition)

Consider a solenoid of N turns with current I.

The coefficient of proportionality is called inductance, L

$$L = \frac{\Phi_m}{I}$$



The SI unit of inductance is the henry, defined as:

1 henry = 1 H = 1 Wb/A = 1 T m2/A

We also found inductance of a solenoid:

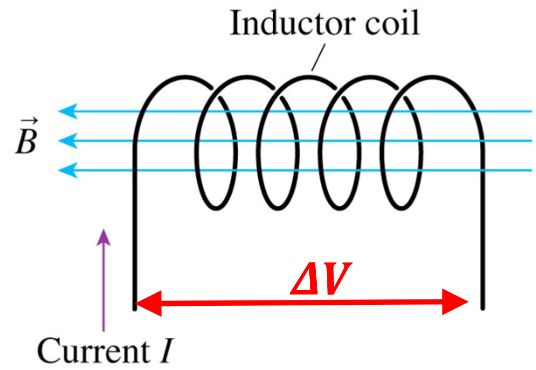
$$L = \frac{\Phi_m}{I} \Rightarrow L = \frac{\mu_0 N^2 A}{l}$$

Energy stored in inductors

$$U_L = \frac{1}{2} L I^2$$

Potential Difference across an Inductor

$$\Delta V = \mathcal{E} = -\frac{d\Phi_m}{dt} = \left| \boldsymbol{L = \frac{\Phi_m}{I}} \quad \boldsymbol{\Phi_m = LI} \right| = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

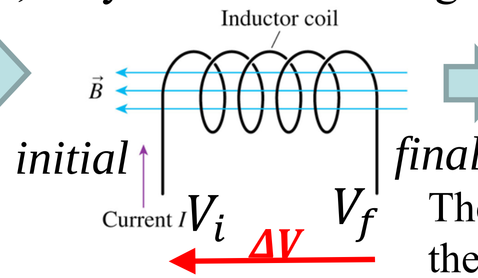


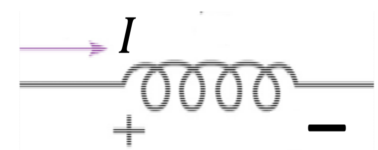
$$\Delta V = \boxed{\mathcal{E} = -L\frac{dI}{dt}}$$ Potential difference across an inductor

Note ! The magnitude of *I* has no effect on ΔV, only the rate of change of *I* counts.

<u>If current increases,</u> $\frac{dI}{dt} > 0 \Rightarrow \Delta V < 0 \Rightarrow$

$$\Delta V = V_f - V_i < 0, so \quad V_f < V_i$$

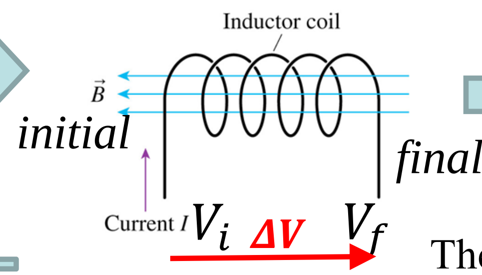


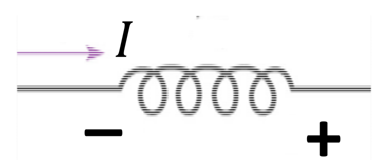


The induced ΔV *decreases if* the current is increasing

<u>If current decreases,</u> $\frac{dI}{dt} < 0 \Rightarrow \Delta V > 0 \Rightarrow$

similar $\quad V_f > V_i$



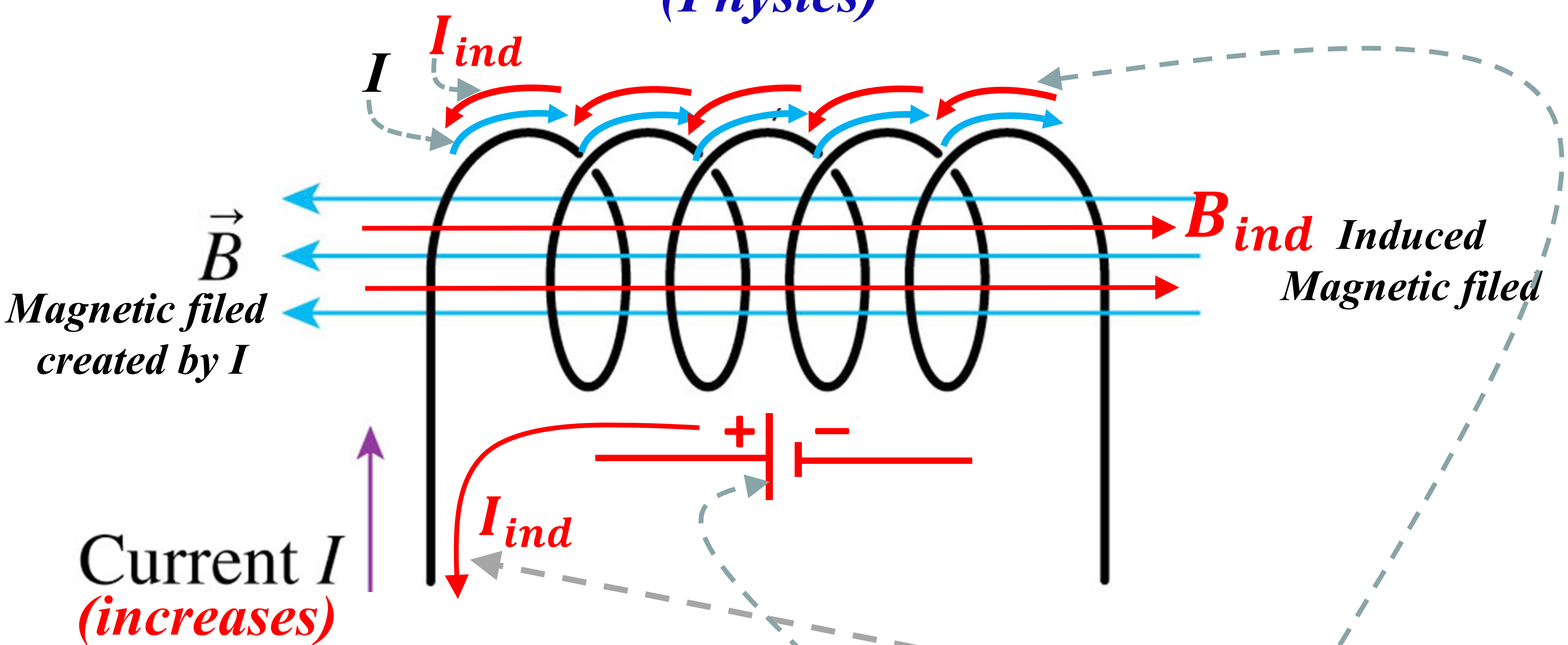


The induced ΔV *increases if* the current is decreasing

If current is constant,*I*=const $\Rightarrow \Delta V = 0$

$\varepsilon = \Delta V$ across a solenoid when the current increase

(Physics)

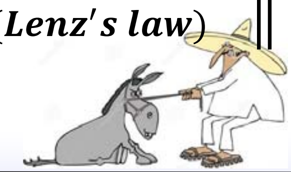


$I \nearrow \Rightarrow B \nearrow \Rightarrow \Phi_B \nearrow \Rightarrow B_{ind} \uparrow\downarrow B_{ext}$ $\Rightarrow$ I_{ind} *is in the opposite direction to a direction of I*

B_{ind} *opposes* B_{ext} *inside the loop (Lenz's law)*

This is an "imaginary battery" which can create this induced current

Thus $V_i > V_f$ $\Rightarrow$ $\Delta V < 0$



ConcepTest 1 ΔV Inductor

- Which current is changing more rapidly?

A. Current I_1

B. Current I_2

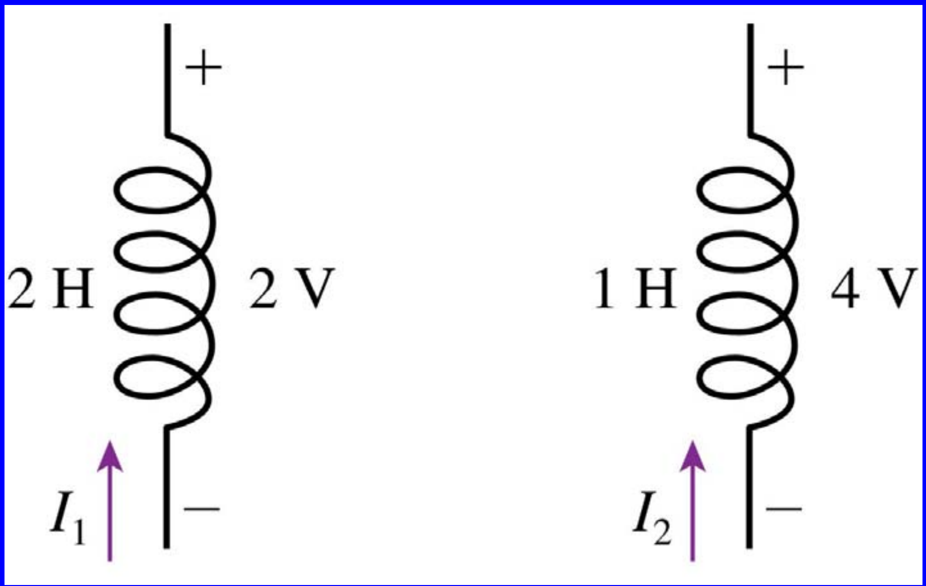
C. They are changing at the same rate

D. Not enough information to tell

$$\Delta V = -L\frac{dI}{dt}$$

$$\left(\frac{dI}{dt}\right)_1 = -\frac{\Delta V_1}{L_1} = -\frac{2V}{2H}$$

$$\left(\frac{dI}{dt}\right)_2 = -\frac{\Delta V_2}{L_2} = -\frac{4V}{1H}$$



Maxwell's equations

Let's revisit Ampere's Law *a straight wire with current I*

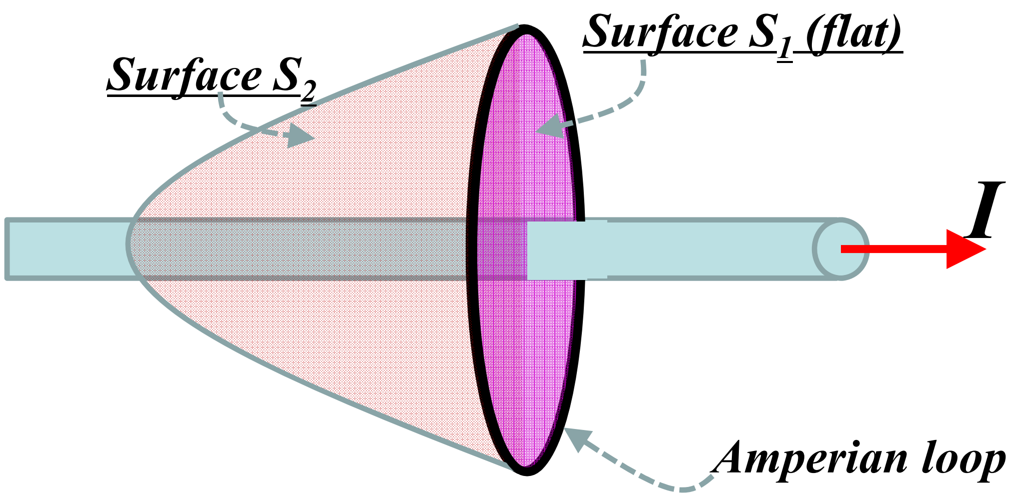
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

The line integral of the magnetic field around the curve is given by Ampère's law:

Any closed loop (Amperian loop)

Current which goes through ANY surface enclosed by an amperian loop

Let's consider a straight wire with current I:

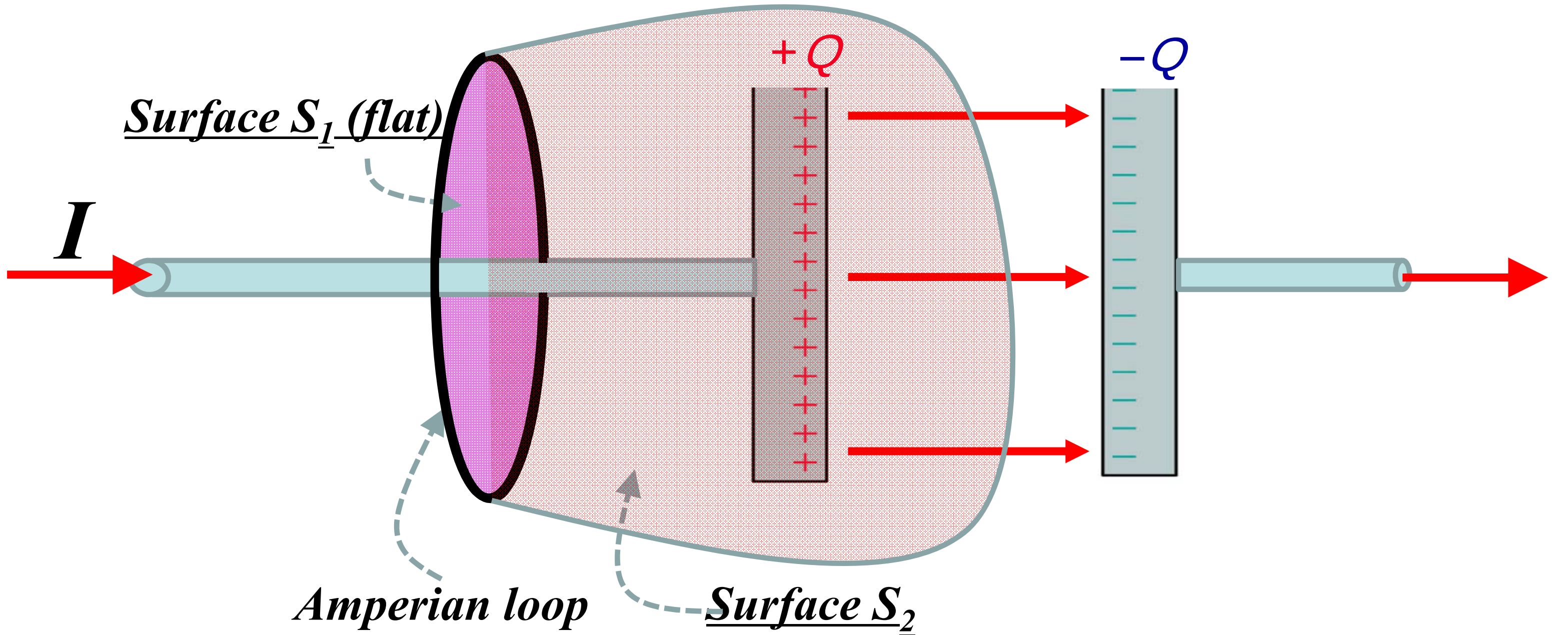


In this example both surfaces (S1 and S2) give us the same enclosed current, as it should be since Ampere's law must work for any possible situation.

Great! Ampere's Law works!

Let's revisit Ampere's Law for *current I and a capacitor*

Let's consider a wire with current I and a capacitor:



Let's apply Ampere's law for both surfaces (S1 and S2):

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow I_{in} = I \Rightarrow \oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

Amperian loop *Surface S_1 (flat)*

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{in} \Rightarrow I_{in} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{s} = 0$$

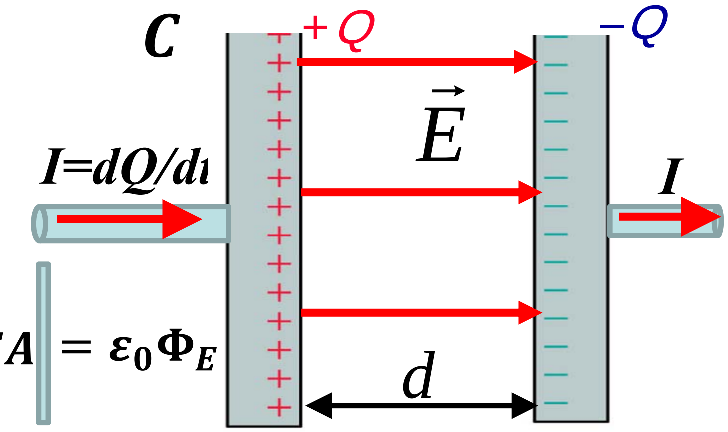
Amperian loop *Surface S_2 (curved)*

The LH sides are the same, but the RH sides are different!!??
Something is missing in Ampere's law.
So!
Ampere's Law needs to be adjusted!

Displacement current/ Ampere-Maxwell Law

Let's get somehow an additional term with units of current and use it to generalize Ampere's Law

$$C \stackrel{\text{def}}{=} \frac{Q}{\Delta V_C}$$



$$Q = C\Delta V = \left| \begin{matrix} C = \varepsilon_0 \frac{A}{d} \\ \Delta V = Ed \end{matrix} \right| = (\varepsilon_0 \frac{A}{d})Ed = \varepsilon_0 EA = \left| \Phi_E = EA \right| = \varepsilon_0 \Phi_E$$

But we need something which has units of current. So let's take a derivative:

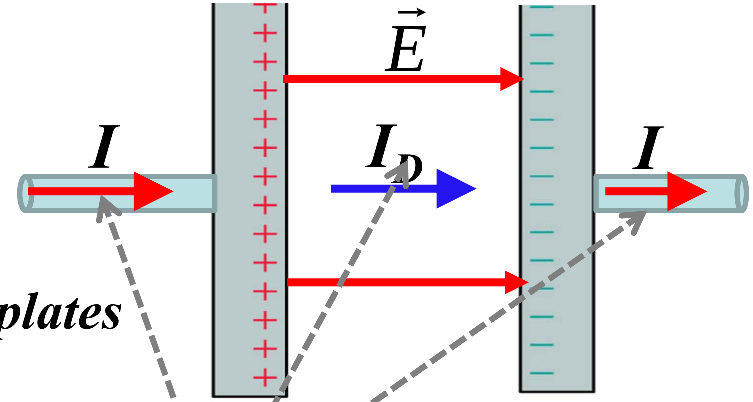
$$I = \frac{dQ}{dt} = \frac{d(\varepsilon_0 \Phi_E)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell interpreted as being equivalent current and called it

a Displacement current $I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{in} + I_D) = \mu_0 (I_{in} + \varepsilon_0 \frac{d\Phi_E}{dt})$$ ***Ampere-Maxwell Law***

Displacement current



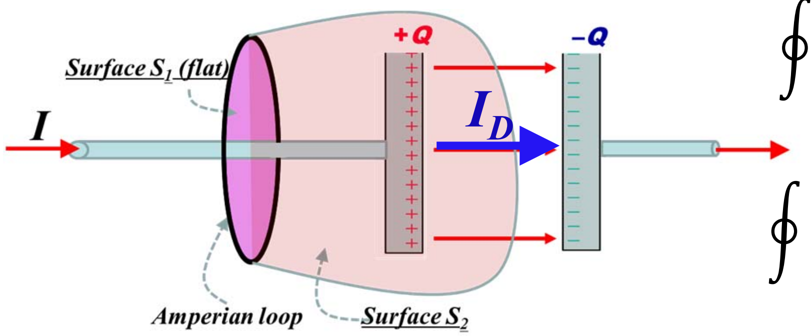
Displacement current $$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

1) The displacement current is only between the plates since $\Phi_E = EA$ *is zero outside*

2) The way I_D was introduced allows us to say that numerically I_D=I (real current in the wire charging the capacitor). In some sense "current" is conserved all the way through the capacitor

3) I_D is not a flow of charge. It is equivalent to a real current in that it creates the same magnetic field

Let's apply Ampere-Maxwell Law for the "capacitor system"



$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I_{in}+I_D) = \left\| \begin{matrix} I_{in} = I \\ I_D = 0 \end{matrix} \right\| = \mu_0\, I$$

Amperian Surface S_1

$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I_{in}+I_D) = \left\| \begin{matrix} I_{in} = 0 \\ I_D = I \end{matrix} \right\| = \mu_0\, I$$

Amperian Surface S_2

Now it works. Each surface gives us the same answer as it should be.

Induced Magnetic Field

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(I_{in} + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Thus, the magnetic field B can be generated by:

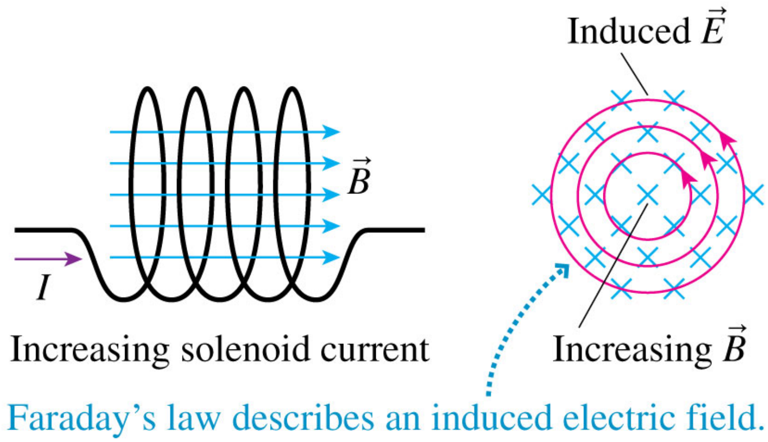
1) ***An ordinary electric current, I_{in}***
2) ***Changing electric flux (particularly, changing electric field)***

Another amazing thing!!!

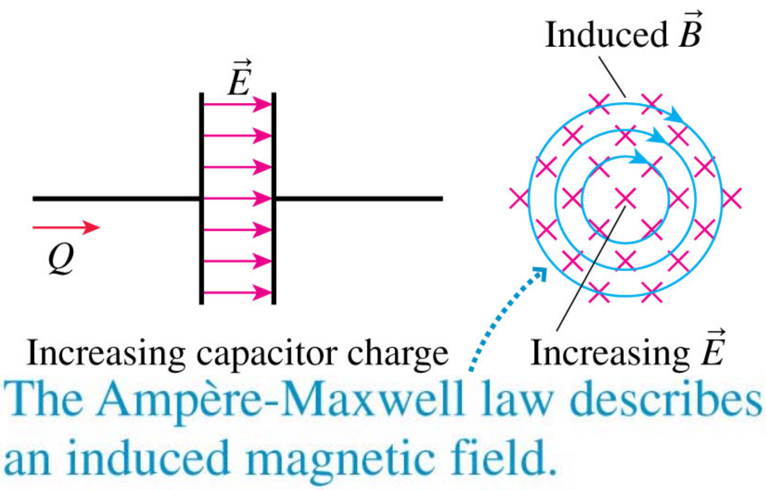
Changing electric field inside a capacitor produces a magnetic field

Induced Fields

- ***An increasing solenoid current causes an increasing magnetic field, which induces a circular electric field.***



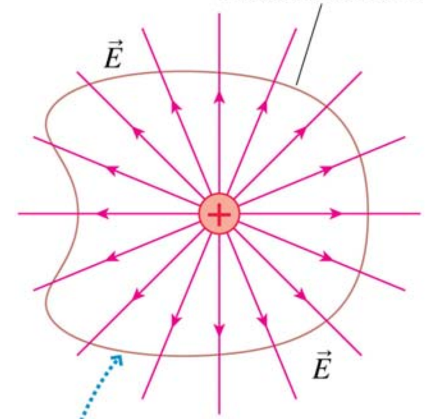
- ***An increasing capacitor charge causes an increasing electric field, which induces a circular magnetic field.***



The last (4th) Maxwell's equation

Gauss's Law for Magnetic Fields

Gauss's law for the electric field says that for any closed surface enclosing total charge Q_{in}, the net electric flux through the surface is:

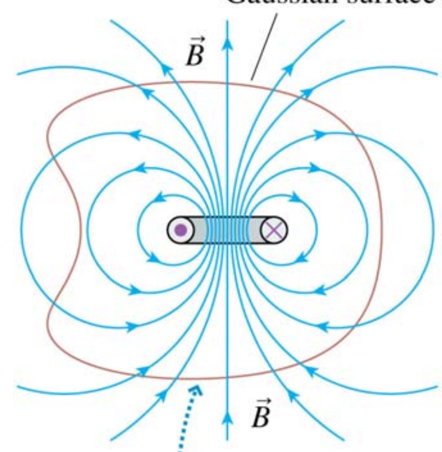


$$(\Phi_e)_{\text{closed surface}} = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

There is a similar equation for a magnetic flux

Magnetic field lines form continuous curves; every field line leaving a surface at some point must reenter it at another.

Gauss's law for the magnetic field states that the net magnetic flux through a closed surface is *zero*:



$$(\Phi_m)_{\text{closed surface}} = \oint \vec{B} \cdot d\vec{A} = 0$$

Maxwell's Equations

Electric and magnetic fields are described by the four **Maxwell's Equations**:

the physical meaning

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad \text{Gauss's law}$$

An electric field is produced by a charge

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's law for magnetism}$$

No magnetic monopoles

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt} \quad \text{Faraday's law}$$

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_{\text{e}}}{dt} \quad \text{Ampère-Maxwell law}$$

A magnetic field is produced by a changing electric field or by a current

In addition to Maxwell's equations, which describes the fields, a fifth equation is needed to tell us how matter responds to these fields:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{(Lorentz force law)}$$

There are a total of 11 fundamental equations describing classical physics:

1. Newton's first law
2. Newton's second law
3. Newton's third law
4. Newton's law of gravity

Physics I

5. Gauss's law
6. Gauss's law for magnetism
7. Faraday's law
8. Ampère-Maxwell law
9. Lorentz force law

Physics II

10. First law of thermodynamics
11. Second law of thermodynamics

Physics III

What you should read

Chapter 34 (Knight)

Sections

- ***34.1 (skip)***
- ***34.2***
- ***34.3***
- ***34.4***

Thank you

See you on Monday